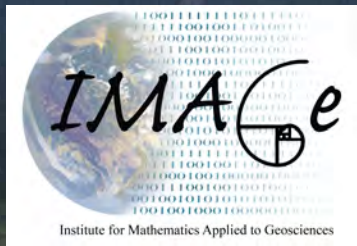


# regionalClimateInformatics

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*National Science Foundation*

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# Credits

- Steve Sain, Tamra Greasby, NCAR
- Dorit Hammerling, SAMSI
- Soutir Bandyopadhyay, Lehigh
- Finn Lindgren, U Bath, UK
- James Gattiker, LANL

# Outline

- Regional Climate simulation and NARCCAP
- A tool for spatial analysis.
- Changes in the seasonality for future climate

## Spatial data:

Given  $n$  pairs of observations  $(x_i, y_i)$ ,  $i = 1, \dots, n$

$$y_i = g(x_i) + \epsilon_i$$

$\epsilon_i$ 's are random errors and  $g$  is an unknown, smooth realization of a Gaussian process.

*Estimate  $g(x)$*

*Quantify the uncertainty of the estimate ...*

**Statistical perspective: You need a model!**

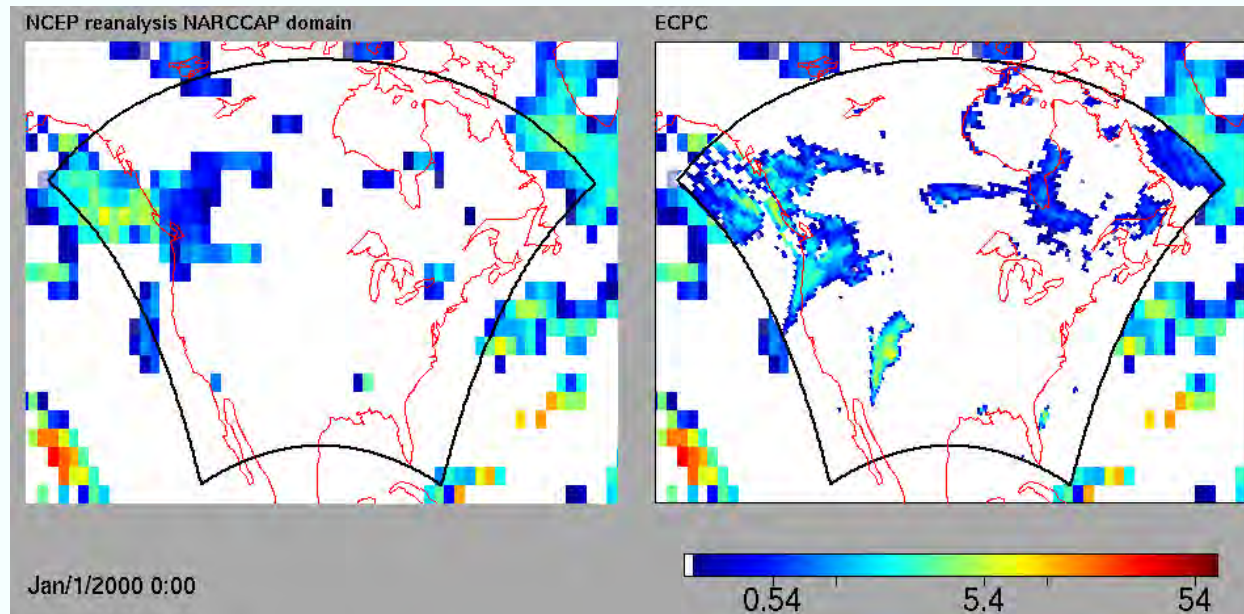
# A climate model grid box (?)



# An approach to Regional Climate

- Nest a fine-scale weather model in part of a global model's domain.

Regional model simulates higher resolution weather based on the global model for boundary values and fluxes.



A snapshot from the 3-dimensional RSM3 model (right) forced by global observations (left)

- Consider different combinations of global and regional models to characterize model uncertainty.

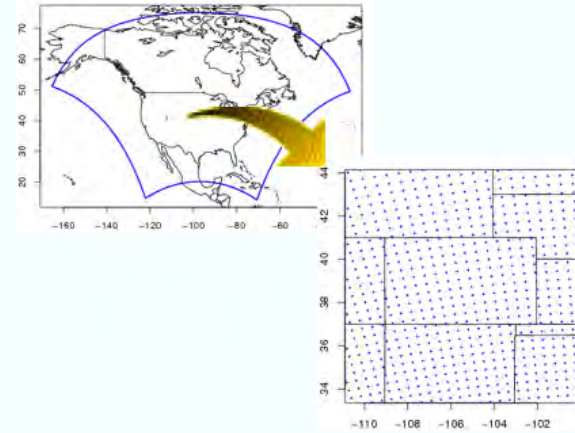
# NARCCAP – the design

4GCMS × 6RCMs:

12 runs – balanced half fraction design

Global observations × 6RCMs

X High resolution global atmosphere



GLOBAL FORCING	REGIONAL MODELS						
	MM5I	WRF	HADRM	REGCM	RSM	CRCM	time slice
GFDL			●	●	○		X
HADCM3	○		●		●		
CCSM	●	■				■	X
CGCM3		■		●		■	
Reanalysis	●	●	●	●	●	●	

*A designed experiment is amenable to a statistical analysis and can contain more information.*

*But just 2-d temperatures fields are 72Gb of data.*

# Climate change

How will the seasonal cycle for temperature change in the future?



# The goals:

- Estimate  $g(x)$  based on the observations
- Quantify the uncertainty in the estimate.
- Handle larger spatial data sets in a interactive mode

$\{\Phi_j\}$ :  $m$  basis functions

$$g(x) = \sum_j \Phi_j(x) c_j$$

*A linear model:*

$$\mathbf{y} = \Phi \mathbf{c} + \boldsymbol{\epsilon}$$

*Random effects:*

$$\mathbf{c} \sim MN(0, \boldsymbol{\rho} \mathbf{P}) \text{ and } \boldsymbol{\epsilon} \sim MN(0, \sigma^2 \mathbf{I})$$

# Key ideas for large data sets

- Inverse of  $P$  chosen to be sparse.
- Basis functions have compact support.
- Still have a useful spatial model!

# The estimate

*Find  $c$  by:*

*Ridge regression/ conditional expectation/BLUE/ Posterior mean*

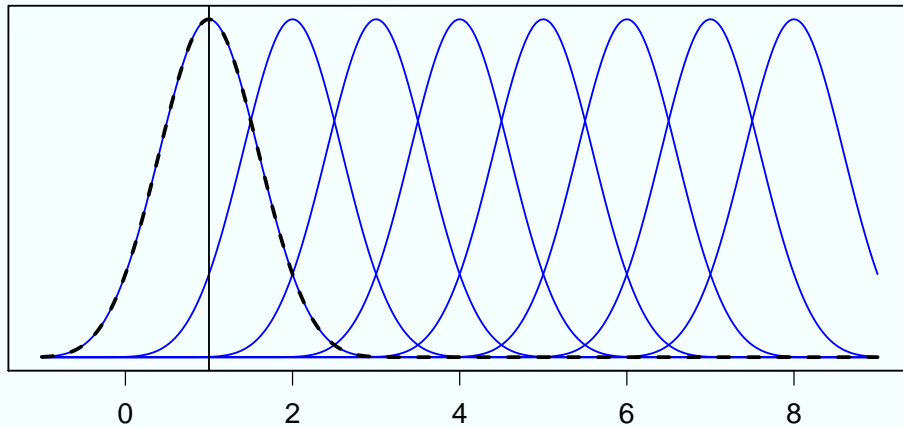
$$\hat{g}(x) = E[g(x)|y, P] = \sum_{k=1}^n \hat{c}_k \Phi_k(x)$$

$$\hat{c} = \left( \Phi^T \Phi + \lambda P^{-1} \right)^{-1} \Phi^T y, \quad \lambda = \sigma^2 / \rho$$

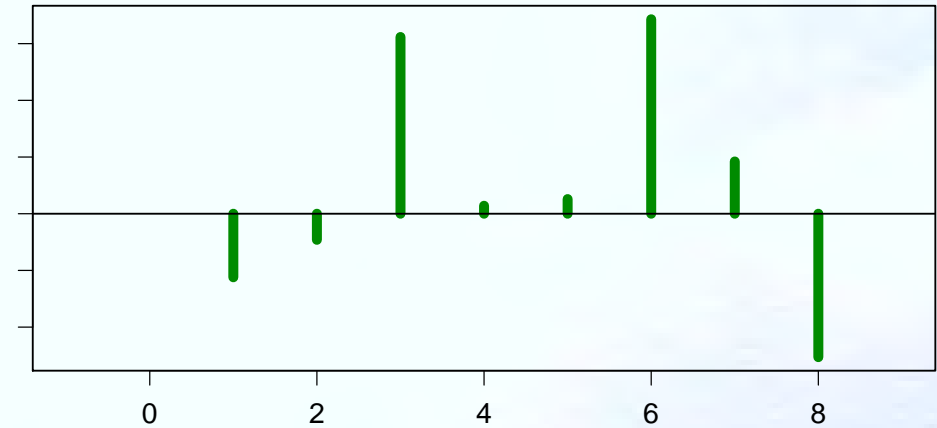
$\Phi^T$ ,  $\Phi^T \Phi$ ,  $P^{-1}$  are sparse.

# A 1-d cartoon ...

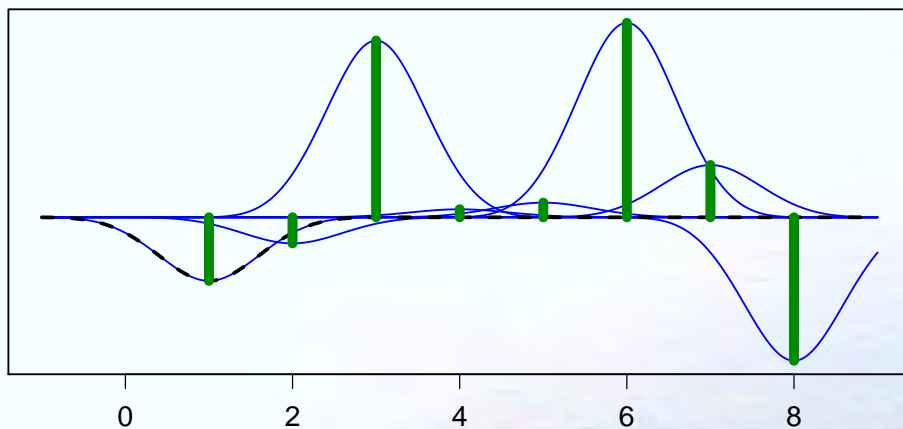
8 basis functions



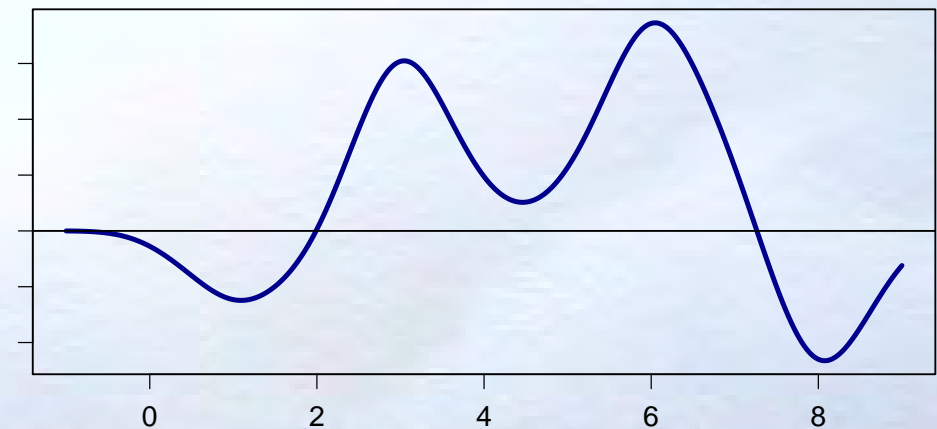
8 (random) weights



weighted basis

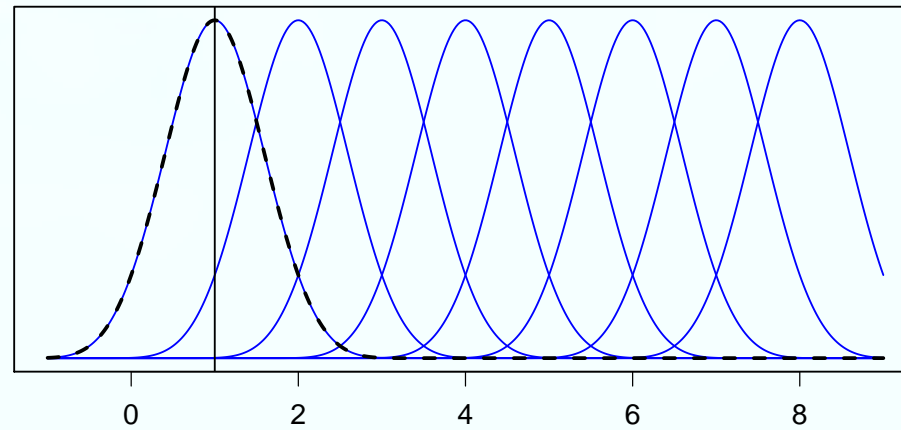


Random curve

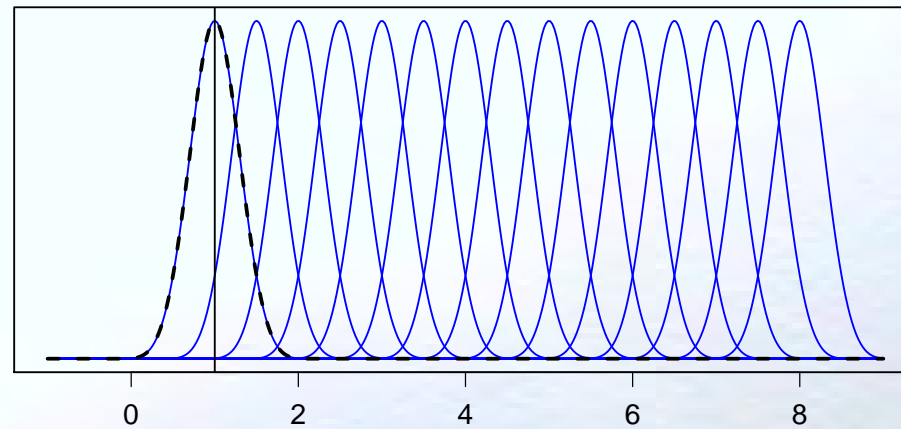


# A Multiresolution

8 basis functions

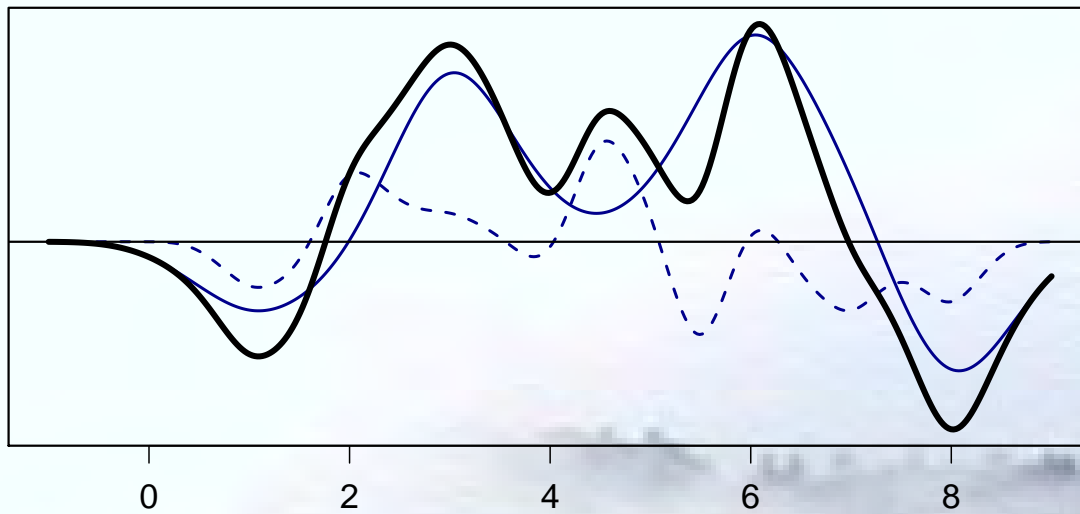
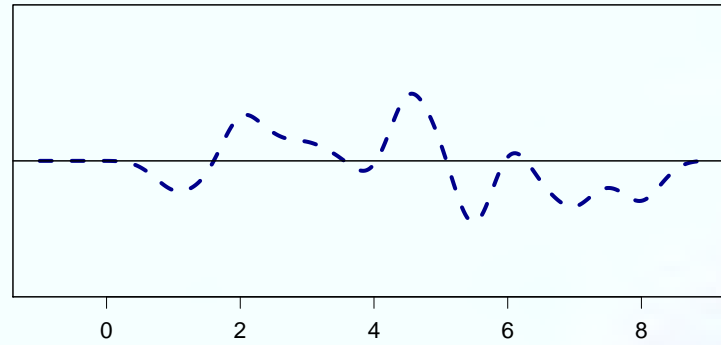
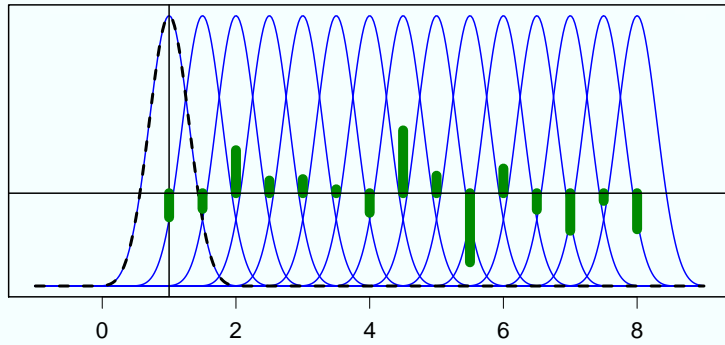
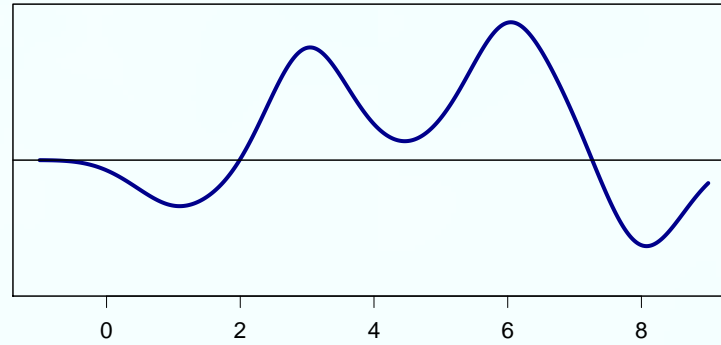
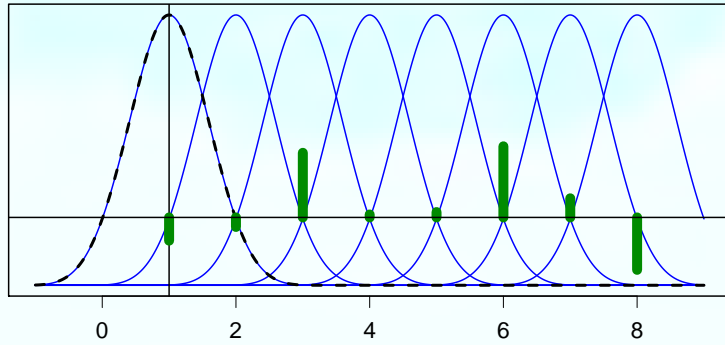


16 basis functions



⋮

# Adding them up



# A recipe for $P^{-1}$

Recall:  $g(x) = \sum_j \Phi_j(x) c_j$

$c$  at each resolution level is a Markov random field:

$$(4 + \kappa^2)c_j - \sum_{l \in \mathcal{N}} c_l = e_j, \quad Hc = e$$

$\{e_j\}$  are uncorrelated  $N(0,1)$  and  $\mathcal{N}$  is 4 nearest neighbors.

*Weights in lattice format:*

$$\begin{array}{cccccc} \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & & -1 & \cdot & \cdot \\ \cdot & -1 & (4 + \kappa^2) & -1 & \cdot & \cdot \\ \cdot & \cdot & & -1 & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot \end{array}$$

**Precision matrix for  $c$  is sparse:  $P = (H^T H)^{-1}$**

# *Back to climate data*



# Climate change

**How will the seasonal cycle for temperature change in the future?**

$g(x, t)$  = Change in mean temperature at location  $x$  and at  $t$  in the year.

# Back to NARCCAP

- A  $2 \times 2$  subset of NARCCAP (4 global/regional combinations) (1971- 2000), (2040-2071) A2 scenario.
- (Future - Present) seasonal cycle expand in 4 principle components ... gives 4 coefficient spatial fields for each model.
- Approximately 8000 spatial locations

*difference in seasonal cycle:*

$$g(\mathbf{x}, t) = a_1(\mathbf{x})\Gamma_1(t) + \dots + a_4(\mathbf{x})\Gamma_4(t)$$

*Principle Components/EOFs:*  $\Gamma_1, \dots, \Gamma_4$

*Amplitudes/coefficients:*  $a_1, \dots, a_4$

- There are four spatial fields to estimate.

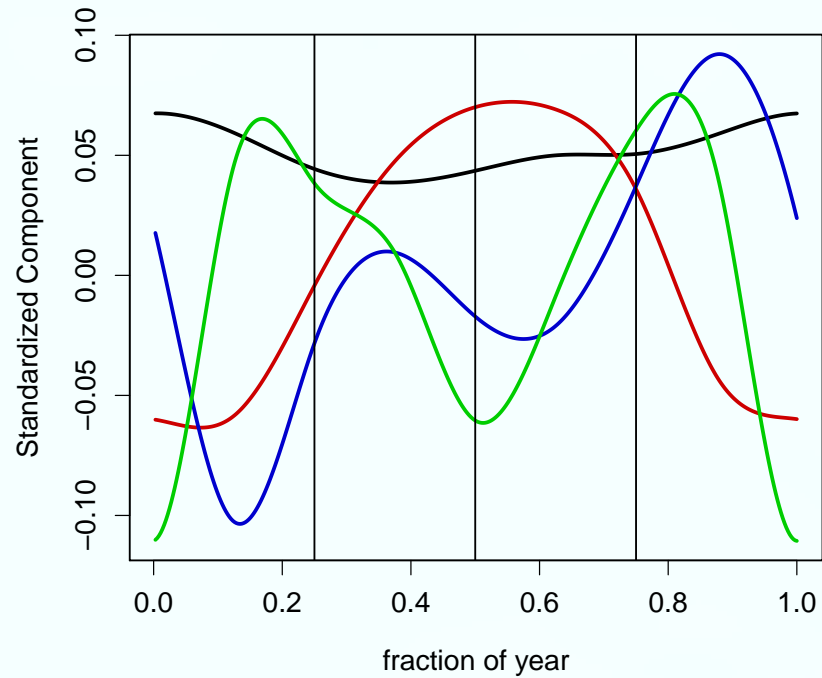
# More on the representation

Seasonal PCs  
(future - present)

$\Gamma_1, \dots, \Gamma_4$

NARCCAP region

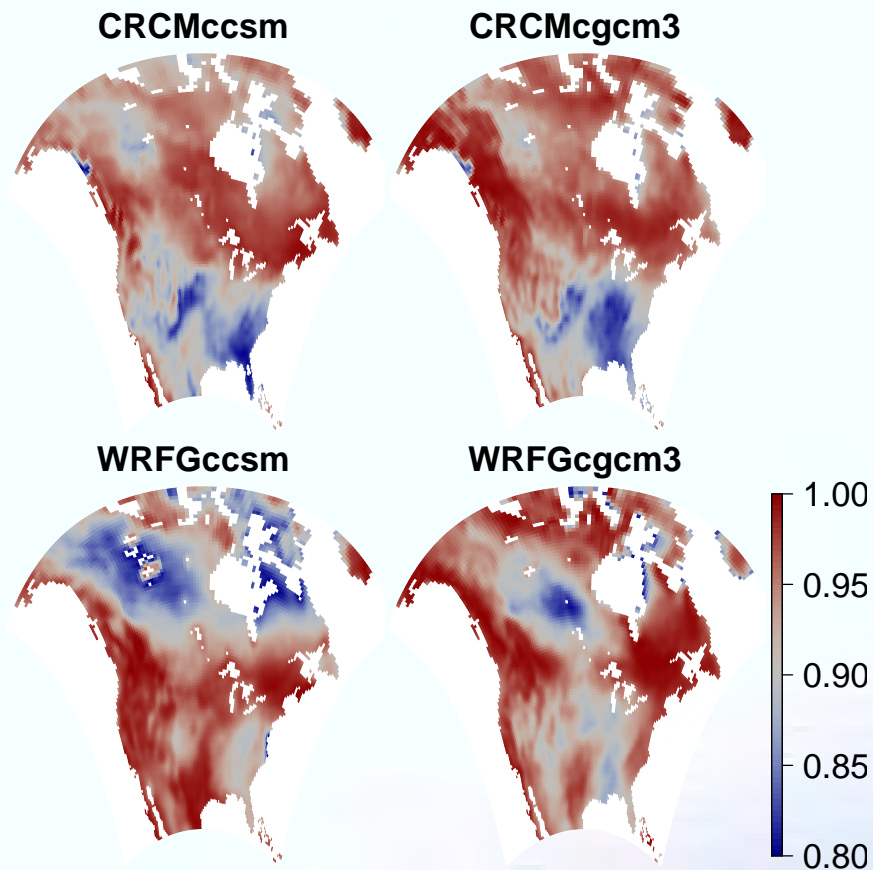
Domain for  $a_1, \dots, a_4$



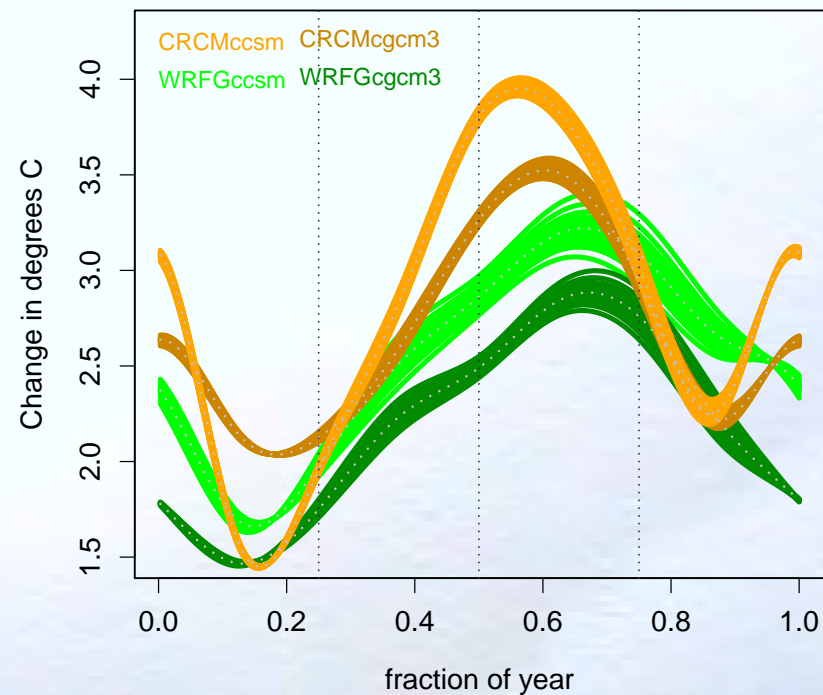
# Results

- Thin plate spline model (1 level  $120 \times 55 \approx 6000$  basis functions)
- $\lambda$  found by MLE (equivalent to sill and nugget)
- Conditional simulation of fields ( facilitates nonlinear statistics)

$R^2$  for first PC



Inference for Boulder grid box



# Summary

- Computational efficiency gained by compact basis functions and sparse precision matrix.
- Flexibility in model to account for nonstationary spatial dependence.
- Multi-resolution can approximate standard covariance families (e.g. Matern)

See `LatticeKrig` *package in R*

# Thank you!

